Written Exam at the Department of Economics winter 2016-17
Advanced Industrial Organization
Final Exam
February 17, 2017
(3-hour closed book exam)

This exam consists of four (4) pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Please answer all five questions

1. *Long-run Relations with Incomplete Contracts.* There are two firms, called U (or Upstream) and D (or Downstream). There is an indivisible product called a "widget". There are two periods. In period 1, U makes an investment I. U can choose I to be any real number between 0 and 9, i.e., $I \in [0, 9]$. In period 2, U may produce either zero or one widget. Obtaining a widget would be worth $v = 12$ to D. The final payoffs are as follows. If U sells the widget to D at price p then D's payoff is $12 - p$ and U's payoff is $p - c(I) - I$, where $c(I)$ is the cost of producing the widget. The investment I is cost-reducing; specifically we assume $c(I) = 12 - 4\sqrt{I}$. If there is no trade, then they get their "disagreement payoffs": D's payoff is 0 and U's payoff is $-I$.

(a) Suppose that they cannot sign any contracts in period 1. Instead, the price p is determined in period 2 according to the Nash bargaining solution. How much will U invest in period 1? Is the equilibrium level of I socially optimal (i.e., first-best)? Show your calculations.

(b) Now suppose that in period 1, before U invests, they can sign a contract that specifies that D has the right to buy a widget in period 2 at a fixed price p (where p is specified in the contract). Does there exist p such that the outcome is first-best? Explain.

2. *Optimal auction.* An indivisible object is up for sale. There are two risk-neutral bidders with independent private values. Bidder 1's valuation v_1 is drawn from a uniform distribution on [0,100]. Bidder 2's valuation v_2 is drawn from a uniform distribution on [50,100]. The seller hires a game-theorist to design an auction that maximizes the expected revenue.

(a) Describe the optimal auction. For different combinations of v_1 and v_2, who gets the object and how much does he pay? Illustrate in a diagram with v_1 on one axis and v_2 on the other. Your reasoning may rely on the results of Bulow-Roberts, without proving or deriving these results.

(b) What is the probability that the good is not sold?

(c) Suppose bidder 1's value is $v_1 = 40$ and bidder 2's value is $v_2 = 55$. Who gets the object, and how much does he pay?

3. *A two-stage game.* Two firms produce a homogeneous good. In stage 1, firm 1 chooses how much to invest in capital equipment. Let k denote the investment level. The investment incurs a sunk cost $\frac{11}{24}k^2$. Firm 1 can choose k to be any (non-negative) real number between 0 and 8, i.e., $k \in [0, 8]$. The investment is observed by firm 2.

In stage 2 there is Cournot competition: firm 1 and firm 2 simultaneously choose quantities $q_1 \geq 0$ and $q_2 \geq 0$. The demand curve is given by the equation $p = 50 - 2Q$, where $Q = q_1 + q_2$. Firm 2's unit production cost is 2, so his profit will be $\pi_2 = (p - 2)q_2$. Firm 1's unit production cost is $2 - k/4$ so his profit will be

$$\pi_1 = pq_1 - \left(2 - \frac{k}{4}\right) q_1 - \frac{11}{24}k^2.$$

Find the subgame perfect equilibrium. Specifically, find the numerical value of k. Show your calculations.

4. *Mixed Logit.* There are two types of consumers in an automobile market, high (H) and low (L). Type H has positive valuation of high horsepower, whereas type L has zero valuation of high horsepower (not everybody likes muscular cars). They respectively account for fractions λ and $1-\lambda$ of the population. There are two alternative cars to choose from, $\mathcal{J} = \{1, 2\}$. Car 1 is low-horsepower, whereas car 2 is high-horsepower. Formally, the utilities of alternative j for the two types are

$$u_{ij}^L = -\alpha p_j + \beta' X_j + \epsilon_{ij}$$
$$u_{ij}^H = -\alpha p_j + \beta' X_j + \gamma horsepower_j + \epsilon_{ij}$$

where p_j is the price, $horsepower_j$ is a dummy variable of high horsepower (thus $horsepower_1 = 0$ for car 1 and $horsepower_2 = 1$ for car 2), and X_j includes other characteristics. The coefficient γ is strictly positive, implying that the high type positively values high horsepower. The coefficients α and β are the same for the two types. Note that for a low-horsepower car, both types derive the same deterministic utility. The idiosyncratic taste ϵ_{ij} is i.i.d. distributed type I extreme value.

(a) Show how to express the market shares of the two products for the two types of consumer, denoted $\sigma_1^H, \sigma_2^H, \sigma_1^L, \sigma_2^L$, using the logit formula for choice probability. (You do not need to show how to derive or prove the Logit formula.)

(b) Suppose a new high-horsepower car, car 3, is introduced into the market ($horsepower_3 = 1$). The new market shares of car 1 and 2 for the two types of consumer are denoted $\hat{\sigma}_1^H$, $\hat{\sigma}_2^H$, $\hat{\sigma}_1^L$, $\hat{\sigma}_2^L$. Assuming p_1 and p_2 do not change, show that the introduction of car 3 proportionally reduces the market shares of cars 1 and 2 more among type H consumers than among type L consumers, i.e.,

$$\frac{\hat{\sigma}_j^H}{\sigma_j^H} < \frac{\hat{\sigma}_j^L}{\sigma_j^L}$$

for $j \in \{1, 2\}$.

5. *Merger Simulation.* Consider a differentiated-product market, where two profit-maximizing firms play a Bertrand-Nash equilibrium of a price-setting game. Firm 1 produces good 1, and firm 2 produces good 2. The demand functions for the two goods have been estimated as follows:

$$q_1 = 11 - 3p_1 + p_2$$

and

$$q_2 = 11 + p_1 - 3p_2.$$

Each firm j has the cost function $C(q_j) = 3q_j$, i.e., a constant unit production cost of 3.

Now the two firms are considering a merger. After the merger, there would be only one profit-maximizing firm, which would produce both good 1 and good 2. The merger would result in efficiency gains: the unit production cost would fall to $3 - x$, where $0 < x < 3$. That is, the cost function would be $C(q_1, q_2) = (3-x)(q_1+q_2)$. Indicate which of the following statements is correct, and carefully justify your answer.

(α) The merger will definitely make consumers worse off.

(β) The merger will definitely make consumers better off.

(γ) There is insufficient information to determine if the merger will make consumers worse off or better off; it depends on the size of the cost-reduction x.

Note: If γ is correct then show for what values of x consumers would be better off.